

# Josephson junction with magnetic-field tunable current-phase relation

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## Abstract

We consider a  $0-\pi$  Josephson junction consisting of asymmetric  $0$  and  $\pi$  regions of different lengths  $L_0$  and  $L_\pi$  having different critical current densities  $j_{c,0}$  and  $j_{c,\pi}$ . If both segments are rather short, the whole junction can be described by an *effective* current-phase relation for the spatially averaged phase  $\psi$ , which includes the usual term  $\propto \sin(\psi)$ , a *negative* second harmonic term  $\propto \sin(2\psi)$  as well as the unusual term  $\propto H \cos \psi$  tunable by magnetic field  $H$ . Thus one obtains an electronically tunable current-phase relation. At  $H = 0$  this corresponds to the  $\varphi$  Josephson junction.

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## I. INTRODUCTION

Recently we proposed<sup>1</sup> to implement a  $\varphi$  Josephson junction (JJ)<sup>2</sup> with magnetic-field tunable current-phase relation (CPR) based on an  $0-\pi$  JJ with the  $0$  and  $\pi$  segments of different length  $L_0 \neq L_\pi$ . This proposal was made keeping in mind YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>-Nb ramp zigzag JJ technology<sup>3,4</sup> (or a similar one<sup>5</sup> with Nd<sub>2-x</sub>Ce<sub>x</sub>CuO<sub>4</sub>-Nb) established recently in our group also<sup>6</sup>. However, in experiment we were more successful<sup>7</sup> in employing superconductor-insulator-ferromagnet-superconductor (SIFS)  $0-\pi$  JJs<sup>8-10</sup>, where the lengths of  $0$  and  $\pi$  segments are equal, but critical current densities  $j_{c,0}$  and  $j_{c,\pi}$  in the  $0$  and  $\pi$  parts are different.

Therefore, in this paper we present a more general theory, which describes an effective  $\varphi$  JJ made of asymmetric  $0$  and  $\pi$  regions of different lengths  $L_0$  and  $L_\pi$  having different critical current densities  $j_{c,0}$  and  $j_{c,\pi}$ .

## II. MODEL

The static sine-Gordon equation that describes the behavior of the Josephson phase  $\phi$  in a  $0-\pi$  JJ is

$$\frac{\Phi_0}{2\pi\mu_0 d_J} \phi'' - j_c(x) \sin \phi = -j. \quad (1)$$

Here  $\mu_0$  is the magnetic flux quantum,  $\mu_0 d_J$  is the specific inductance (per square) of the superconducting electrodes forming the JJ and  $j$  is the bias current density. The prime denotes the partial derivatives with respect to coordinate  $x$ . We assume that the critical current density  $j_c(x)$  has the form of a step-function

$$j_c = j_{c,0} > 0, \quad 0 \leq x \leq L_0, \quad (2)$$

$$j_c = j_{c,\pi} < 0, \quad -L_\pi \leq x < 0. \quad (3)$$

We write the critical current density  $j_c(x)$  as

$$j_c(x) = \langle j_c \rangle [1 + g(x)], \quad (4)$$

where

$$\langle j_c \rangle = \frac{1}{L} \int_{-L_\pi}^{L_0} j_c(x) dx = \frac{1}{L} (j_{c,0} L_0 + j_{c,\pi} L_\pi) \quad (5)$$

is the average critical current density,  $L = L_0 + L_\pi$  is the total length of the junction, and  $\langle g(x) \rangle = 0$ . The function  $g(x)$  is defined as

$$g(x) = \frac{j_c(x)}{\langle j_c(x) \rangle} - 1 \quad (6)$$

that results in

$$g(x) = \begin{cases} g_0, & 0 < x < L_0, \\ g_\pi, & -L_\pi < x < 0. \end{cases} \quad (7)$$

where

$$g_0 = \frac{(j_{c,0} - j_{c,\pi})L_\pi}{j_{c,0}L_0 + j_{c,\pi}L_\pi}; \quad g_\pi = -\frac{(j_{c,0} - j_{c,\pi})L_0}{j_{c,0}L_0 + j_{c,\pi}L_\pi}. \quad (8)$$

Then we divide Eq. (1) by  $|\langle j_c \rangle|$  and normalize the coordinate  $x$  to the Josephson length calculated using  $|\langle j_c \rangle|$ , *i.e.*,

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0 d_J |\langle j_c \rangle|}}. \quad (9)$$

Thus, we obtain a normalized sine-Gordon equation for the phase difference  $\phi(x)$

$$\phi'' - \text{sgn}(\langle j_c \rangle) [1 + g(x)] \sin \phi = -\gamma, \quad (10)$$

where  $\gamma = j/|\langle j_c \rangle|$  is the normalized bias current density. It is worth mentioning that  $\langle j_c \rangle$  can be positive as well as negative. Below, for the same of simplicity, we assume  $\langle j_c \rangle > 0$ .

Thus, Eq. (10) becomes

$$\phi'' - [1 + g(x)] \sin \phi = -\gamma. \quad (11)$$

In the case  $\langle j_c \rangle < 0$  the substitution  $\phi \rightarrow \pi - \phi$  converts Eq. (10) to the same Eq. (11).

We look for a solution of Eq. (11) in the form

$$\phi(x) = \psi + \xi(x) \sin \psi, \quad (12)$$

where

$$\psi = \langle \phi(x) \rangle \quad (13)$$

is a constant *average phase*, while  $\xi(x) \sin \psi$  describes the deviation of the phase from the average value, *i.e.*,  $\langle \xi(x) \rangle = 0$ . Further we assume that the deviation is small, *i.e.*,  $|\xi(x) \sin \psi| \ll 1$ . Then we plug the relation (12) into Eq. (11), expand it in series in  $\xi(x) \sin \psi$ , and keep the terms of zero and first order. We get

$$\xi'' \sin \psi - [1 + g(x)][1 + \xi(x) \cos \psi] \sin \psi = -\gamma. \quad (14)$$

The constant terms (zero order of  $\xi$  in Eq. (14)) are

$$\gamma = \sin \psi + \langle \xi(x)g(x) \rangle \cos \psi \sin \psi. \quad (15)$$

The terms of first order of  $\xi(x)$  in Eq. (14) are

$$\xi'' - g(x) = \{\xi + \xi(x)g(x) - \langle \xi(x)g(x) \rangle\} \cos \psi. \quad (16)$$

Numerical calculations show that the two terms  $\propto \cos \psi$  have an extremely weak effect on solutions of Eq. (16). We neglect these terms and obtain for  $\xi(x)$

$$\xi'' - g(x) = 0. \quad (17)$$

We treat solutions of Eq. (17) by using the matching continuity (at  $x = 0$ ) and boundary (at  $x = -l_\pi \equiv -L_\pi/\lambda_J$ ,  $x = l_0 \equiv L_0/\lambda_J$ ) conditions

$$\xi_\pi(0) = \xi_0(0), \quad \xi'_\pi(0) = \xi'_0(0), \quad (18)$$

$$\xi'_\pi(-l_\pi) \sin \psi = h, \quad \xi'_0(l_0) \sin \psi = h. \quad (19)$$

The applied field  $H$  is normalized by  $H_{c1}/2$ , *i.e.*,

$$h = \frac{2H}{H_{c1}}, \quad H_{c1} = \frac{\Phi_0}{\pi \Lambda \lambda_J}, \quad (20)$$

where  $\Lambda$  is the effective magnetic thickness of the JJ. We integrate Eq. (17) once and obtain

$$\xi'_0(x) = g_0(x - l_0) + \frac{h}{\sin \psi}, \quad 0 < x < l_0, \quad (21)$$

$$\xi'_\pi(x) = g_\pi(x + l_\pi) + \frac{h}{\sin \psi}, \quad -l_\pi < x < 0. \quad (22)$$

The second integration results in

$$\xi_0(x) = g_0 \left( \frac{x^2}{2} - l_0 x \right) + \frac{hx}{\sin \psi} + C, \quad (23)$$

for  $0 < x < l_0$ ,

$$\xi_\pi(x) = g_\pi \left( \frac{x^2}{2} + l_\pi x \right) + \frac{hx}{\sin \psi} + C, \quad (24)$$

for  $-l_\pi < x < 0$ .

The integration constant  $C$  can be obtained using the condition  $\langle \xi(x) \rangle = 0$

$$C = \frac{l_0 - l_\pi}{2} \left( \frac{g_0 l_0 + g_\pi l_\pi}{3} - \frac{h}{\sin \psi} \right). \quad (25)$$

We use Eqs. (7), (23), and (24) and obtain the average  $\langle \xi(x)g(x) \rangle$  in the form

$$\langle \xi(x)g(x) \rangle = \Gamma_0 + \Gamma_h \frac{h}{\sin \psi}, \quad (26)$$

where the coefficients  $\Gamma_0$  and  $\Gamma_h$  are given by

$$\Gamma_0 = -\frac{l_0^2 l_\pi^2}{3} \frac{(j_{c,0} - j_{c,\pi})^2}{(j_{c,0} l_0 + j_{c,\pi} l_\pi)^2}, \quad (27)$$

$$\Gamma_h = \frac{l_0 l_\pi}{2} \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} l_0 + j_{c,\pi} l_\pi}. \quad (28)$$

Using Eqs. (15) and (26) we find the current-phase relation in the form

$$j = \langle j_c \rangle (\sin \psi + \Gamma_0 \sin \psi \cos \psi + h \Gamma_h \cos \psi). \quad (29)$$

It is worth noting that there is a simple relation between the coefficients  $\Gamma_0$  and  $\Gamma_h$ . Indeed, it follows from Eqs. (27) and (28) that

$$\Gamma_0 = -\frac{4}{3} \Gamma_h^2. \quad (30)$$

In the case of equal lengths of 0 and  $\pi$  parts ( $l_0 = l_\pi = l/2$ ) we find

$$\Gamma_0 = -\frac{l^2}{12} \left( \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}} \right)^2, \quad \Gamma_h = \frac{l}{4} \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}}. \quad (31)$$

The energy  $U(\psi)$  corresponding to the current-phase relation (29) is given by

$$U(\psi) = \langle j_c \rangle \left( 1 - \cos \psi + h \Gamma_h \sin \psi + \frac{\Gamma_0}{2} \sin^2 \psi \right). \quad (32)$$

### III. CONCLUSIONS

We have extended our previous results<sup>1</sup> to the case of arbitrary critical current densities  $j_{c,0} \neq j_{c,\pi}$  more relevant for experiment<sup>7</sup>. The dependence (29) of the CPR on the phase and applied field is the same as in our previous study<sup>1</sup>. The difference is in the formulas (28) for  $\Gamma_0$  and  $\Gamma_h$ .

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